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Higgs boson cosmology

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The discovery of the Standard Model Higgs boson opens up a range of speculative cosmological scenarios, from the formation of structure in the early universe immediately after the big bang, to relics from the electroweak phase transition one nanosecond after the big bang, on to the end of the present-day universe through vacuum decay. Higgs physics is wide-ranging, and gives an impetus to go beyond the Standard Models of particle physics and cosmology to explore the physics of ultra-high energies and quantum gravity.

Keywords: Higgs boson, cosmology, inflation, symmetry breaking.

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1. Introduction

The discovery of the Higgs boson [1, 2] has completed the list of fermions (quarks and leptons), vector bosons (photons, gluons, W and Z) and the single scalar boson that make up the Standard Model of particle physics [3]. The Higgs has given us the first glimpse of a particle associated with a fundamental scalar field. This has a special significance to cosmologists, because so many of our modern theories of the early universe depend on the unique features of scalar field physics.

We know that universe was once incredibly hot. As the universe cooled, it most likely went through a sequence of phase transitions. The Standard Model Higgs field has a special association with an electroweak phase transition, which took place at a temperature equivalent to an energy scale of 163 GeV around one nanosecond after the big bang. If we are fortunate, phase transitions like this can leave behind signals or relics which give us information about physics beyond the Standard Model.

There is a great deal of interest in a phase transition which happened far earlier, at a time around 10^{-35} s after the big bang. According to the inflationary scenario, the universe was dominated then by the vacuum energy of a scalar field, causing it to undergo a period of exponential expansion [4–6]. Originally, the idea was that this field would be the Higgs field in a Grand Unified Theory uniting the strong, weak and electromagnetic forces. It is easier for the exponential expansion to get going when the energy scale of the phase transition is very high, and this fits in well with the idea of Grand Unification at around 10^{15} GeV.

Inflationary models lead to a natural explanation for the origin of the large scale structure of the universe today. We measure this large scale structure primarily through observations of cosmic microwave background (CMB) radiation and through galaxy surveys. The CMB gives us a snapshot

of the universe from the time when it first became transparent, around 370,000 yr after the big bang. Galaxy surveys (together with distortions of the CMB) probe the large scale structure from just after the origin of the CMB up until the present day. In inflationary models, small quantum or thermal fluctuations from the era of inflation are frozen into the geometry of spacetime. These fluctuations are the cause of the intensity fluctuations in the cosmic microwave background. Different models of inflation, with different dependence of the vacuum energy on the scalar field, give different predictions for the cosmic microwave background fluctuations and allow us to test models of inflation.

The large vacuum energy needed for inflation can arise in models where large fields correspond to large vacuum energies, provided that the universe started out with a large-magnitude scalar field. This is achieved in the chaotic inflationary scenarios, where the field starts out randomly distributed throughout space [7]. By chance, some small region has a large magnitude field and this region inflates. Eventually, with enough inflation, this region grows to encompass our entire observable universe. Once we get into the setting of these chaotic scenarios, then even a field normally associated with energy scales well below 10^{15} GeV, like the Standard Model Higgs field, can drive inflation. This is the main topic covered in section 3.

Finally, we might enquire whether all the phase transitions are in the distant past, or whether we live in a metastable state, or ‘false vacuum’, which could decay into a new vacuum state [8]. For the observed Higgs mass around 125 GeV, the vacuum structure of the Higgs field in the Standard Model becomes rather sensitive to couplings to other massive particles. One possible configuration is a metastable state with the Higgs field having a value around 246 GeV and a true vacuum with the Higgs field close to the Planck scale 2.4×10^{18} GeV. This would, of course, be very susceptible to the effects of new physics beyond the Standard Model.

Decay of our ‘false’ vacuum state would be catastrophic, since it would change the masses and interactions of all the elementary particles. The fact that our vacuum has survived 13.8 Gyr since the big bang is evidence that the decay rate must be negligible. Vacuum decay is a quantum tunnelling process which is exponentially suppressed, so very long lifetimes for metastable states are not unusual. On the other hand, everyday phase transitions are often triggered by some kind of seed, a speck of ice or dust in a cloud for example causing water to condense into droplets. In the last section we shall report on recent results which consider whether a tiny black hole could act as a nucleation seed and trigger the decay of our vacuum state.

The theory of elementary particles is greatly simplified by using a special set of units in which $\hbar = c = 1$. The fundamental unit is then the GeV = 10^9 eV = 0.1602 J (S.I.). In these units masses are given in GeV, distances in $\text{GeV}^{-1} = 0.1973$ fm (S.I.). The gravitational constant is replaced by the reduced Planck mass, $M_p = (8\pi G)^{-1/2} = 2.435 \times 10^{18} \text{GeV} = 4.3415 \times 10^{-9}$ kg (S.I.).

2. The Higgs field

We start with an overview of some of the essential features of Higgs field theory in the part of the Standard Model which unites weak and electromagnetic forces: the Weinberg-Salam model[9, 10]. This model has an $SU(2) \times U(1)$ local symmetry group.

The unified Higgs field consists of two complex scalar fields arranged into a doublet $\mathbf{H}(\mathbf{x}, t)$. The Higgs field is a scalar under spatial rotations, but the components mix under the $SU(2) \times U(1)$ symmetry group. The energy density of a stationary Higgs field configuration defines the Higgs potential $V(\mathbf{H})$,

$$V(\mathbf{H}) = V_0 - \mu^2 \mathbf{H}^\dagger \mathbf{H} + \lambda (\mathbf{H}^\dagger \mathbf{H})^2. \quad (1)$$

Adding the constant V_0 has no effect on the particle physics, but it becomes important for cosmology because all forms of energy affect the expansion of the universe.

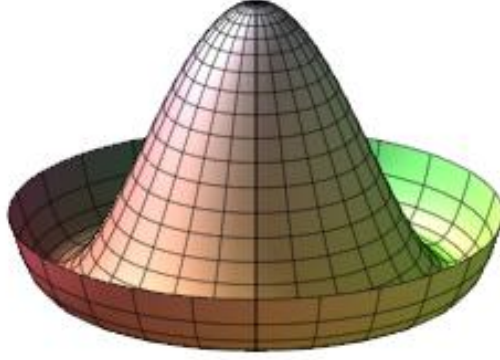


Figure 1. The famous ‘Mexican hat’ shape for the Higgs potential. The potential is plotted vertically and solid lines indicate constant (ϕ, α) parameters for the Higgs field, leaving two angles β and γ which are not shown. The Higgs vacuum corresponds to a field lying somewhere along the brim of the hat. The potential there has to be very close to zero, otherwise it would produce a large cosmological constant.

In exactly the same way in which we might represent a vector by its magnitude and direction, we can represent the Higgs field by a magnitude ϕ and rotation angles α , β and γ ,

$$\mathbf{H} = \frac{1}{\sqrt{2}} \mathbf{U} \begin{pmatrix} 0 \\ \phi \end{pmatrix}, \quad \mathbf{U} = \begin{pmatrix} \cos \alpha + i \sin \alpha \cos \beta & ie^{-i\gamma} \sin \alpha \sin \beta \\ ie^{i\gamma} \sin \alpha \sin \beta & \cos \alpha - i \sin \alpha \cos \beta \end{pmatrix}. \quad (2)$$

The potential has the ‘Mexican hat’ shape shown in figure 1, with the minimum of the potential at $\phi = v = \mu^2/\lambda$. There is no dependence of the potential on the angles α , β and γ .

The value of the potential at the minimum, where the Higgs field sits today, is tightly constrained by cosmology. The potential acts like a cosmological constant term in the gravitational field equations and leads to exponential expansion. Whilst the current observational evidence is consistent with the universe undergoing exponential expansion, the scale is tiny today compared to any energy scale relevant to the Higgs boson. Taking into account the zero potential at the minimum, the Higgs potential can be expressed succinctly as

$$V(\phi) = \frac{1}{4} \lambda (\phi^2 - v^2)^2. \quad (3)$$

In quantum theory, the Higgs field becomes a quantum operator with the vacuum expectation value set by the minimum of the potential, $\langle \phi \rangle = v$. The Higgs boson corresponds to an excitation of the

vacuum state with mass

$$m_H = \sqrt{V''(v)} = \lambda^{1/2}v, \quad (4)$$

which the Large Hadron Collider has determined to be 126 GeV.

The essential feature of the Higgs mechanism is what happens to the fields α , β and γ . Although the Higgs boson is a neutral particle, the unified Higgs field of the Weinberg-Salam model is charged. It couples to a $U(1)$ vector potential \mathbf{B} with strength g' and a triplet of $SU(2)$ vector potentials \mathbf{W}_1 , \mathbf{W}_2 , \mathbf{W}_3 , with strength g . The fields α , β and γ are absorbed into a field redefinition of these vector potentials, $\mathbf{W}_i \rightarrow \mathbf{W}'_i$. This results in three massive vector fields W_{\pm} and Z , with masses

$$m_W = m_Z \cos \theta_W = \frac{gv}{2} = 80.4 \text{ GeV}, \quad (5)$$

where $\tan \theta_W = g'/g$. One final combination of the vector fields remains massless, and defines the electromagnetic potential \mathbf{A} ,

$$\mathbf{A} = \mathbf{B} \cos \theta_W + \mathbf{W}'_3 \sin \theta_W, \quad (6)$$

where \mathbf{W}'_3 is the re-defined \mathbf{W}_3 . The elementary electromagnetic charge is related to the $SU(2)$ coupling by $e = g \sin \theta_W$.

All of the above is subject to the addition of quantum corrections to the potential, the couplings and the relations between the couplings and the masses [10]. As a result, the effective values of the coupling constants become ‘running’ coupling constants which depend on an energy scale. The gauge couplings g and g' , for example, run at different rates and appear to converge roughly to a common value at an energy scale around 10^{15} GeV, suggesting that the symmetry becomes enlarged into a unified symmetry group [11]. The Higgs self coupling also runs, to low values at high energies, which is something we shall return to later.

3. The inflationary Higgs

The power of modern cosmology is amply demonstrated in the way it removes the possibility that the Standard Model Higgs field could be the field responsible for inflation. We shall see that the Higgs field is perfectly capable of producing inflation, but that resulting large-scale structure of the universe is at odds with the high precision observations of the large scale structure of the universe which are now possible.

To have inflation driven by the Higgs field, the Higgs potential energy density has to come to dominate over everything else, and it has to hang around for long enough to drive the universe into exponential expansion. The theory of inflation gives a necessary condition for this to happen in terms of a ‘slow-roll’ parameter ϵ_V [12],

$$\epsilon_V = \frac{M_p^2}{2} \left(\frac{V'}{V} \right)^2 < 1. \quad (7)$$

At large values of $\phi \gg v$, the Higgs potential (3) has a quadratic behaviour,

$$V(\phi) \approx \frac{1}{4} \lambda \phi^4, \quad (8)$$

Thus $\epsilon_V \approx 8M_p^2/\phi^2$, and inflation could in principle take place for $\phi > \sqrt{8}M_p$. Since the Higgs field contributes to particle masses, this runs the risk of putting some of these masses beyond the

Planck scale, introducing quantum gravity effects. Any discussion of Higgs inflation is subject to some degree uncertainty for this reason.

The most remarkable feature of the inflationary scenario is the generation of primordial density fluctuations from quantum fluctuations during inflation [13–17]. The fluctuations can be separated into Fourier modes which track the expansion of the universe, so that a mode with wave number k grows in wavelength with the growth of the universe, from around 10^{-28} m during the inflationary era to present-day cosmological scales measured in megaparsecs (Mpc). The evolution of the amplitude of these fluctuations into the temperature fluctuations seen in the Cosmic Microwave Background involves relatively low energy physics and is believed to be well-understood [12]. It is therefore possible to reconstruct features of the primordial fluctuations from the CMB, and other large-scale structure observations.

Inflationary fluctuations which influence the CMB come in two types: scalar and tensor. Scalar modes describe the fluctuations in the energy density and the tensor modes are gravitational waves. Both types of fluctuation have an effect on the local intensity and polarisation of the CMB. The primordial power spectra of these fluctuations are conventionally parameterised by

$$\mathcal{P}_s = A_s \left(\frac{k}{k_*} \right)^{n_s-1}, \quad (9)$$

$$\mathcal{P}_t = A_t \left(\frac{k}{k_*} \right)^{n_t-1}, \quad (10)$$

where the spectral indices n_s and n_t give the leading order dependence on the wave number k . A typical value used for the pivot scale is $k_* = 0.05 \text{ Mpc}^{-1}$. (The wavelength of the mode k_* is presently around 126 Mpc. To give some idea of scale, a cube of the corresponding size would contain around 10,000 galaxies.) Recent data from the Planck satellite [18, 19], supported by a whole range of other observations, suggest that the scalar fluctuations predominate, with $A_s \approx 2.25 \times 10^{-9}$. The tensor contribution, tracked by the tensor to scalar ratio $r = \mathcal{P}_t/\mathcal{P}_s$, is less than 10% of the total.

The theory of inflationary fluctuations gives us the scalar fluctuation amplitude in terms of the potential and the slows-roll parameter ϵ_V as

$$A_s = \frac{1}{24\pi^2} \frac{V_*}{M_p^4 \epsilon_{V*}}. \quad (11)$$

The wavelength of a fluctuation of wave-number k actually increases with the expansion of the universe. The star on the potential V_* indicates that the potential is evaluated at a time t_* when the wavelength of the expanding fluctuation equals the natural length scale, or Hubble radius, of the inflationary universe. The logarithmic growth of the wavelength between t_* and the end of inflation is an important number, called the ‘number of e-folds’, N_* . This is related to the slow-roll parameter by $N_* \approx 1/\epsilon_{V*} \gg 1$, allowing us to express the amplitude (11), using (7) and (8), in terms of N_* ,

$$A_s = \frac{2}{3\pi^2} \lambda N_*^3. \quad (12)$$

The number of e-folds is set by the requirement that the density fluctuation has to grow in wavelength from the Hubble radius 10^{-28} m at the time t_* during inflation, to 0.05 Mpc today. This requires inflationary models to have N_* between 50 and 60, depending on how the universe evolves after the end of the inflationary era. We know A_s from the size of the temperature fluctuations in the CMB, and combining this with $N_* > 50$ gives an incredibly tight limit on the coupling $\lambda < 0.3 \times 10^{-12}$. We will see in Sect 5 that the Higgs coupling becomes small at high values of the Higgs field due to quantum effects, but even so such a small value seems implausible.

The final, and conclusive, blow to this simplest type of Higgs inflation comes from the ratio of tensor to scalar fluctuations. Inflationary theory gives the tensor amplitude

$$A_t = \frac{2}{3\pi^2} \frac{V_*}{M_p^4}. \quad (13)$$

The ratio of (13) to (11) is therefore

$$r_* = 16\epsilon_{V_*} = \frac{16}{N_*} > 0.27, \quad (14)$$

for $N_* < 60$. This conflicts with the limit $r < 0.1$ obtained from observations of the CMB and the large-scale galaxy distribution.

Besides simple Higgs inflation, this argument shows that any type of inflation based the relativistic wave equation and a quartic potential is in conflict with the observations. The argument has to be reconsidered if there are modifications to the derivative terms in the theory, or coupling to other scalar fields with a different potential. This happens in the variations of Higgs inflation described below. Nevertheless, ruling out simple inflationary models with quartic potentials is a remarkable achievement of observational cosmology.

3.1. *Type I Higgs inflation*

The major omission from the Standard Model of particle physics is gravity. We might expect the elementary particles of the Standard Model to satisfy the basic principles of relativity, and couple to the geometry of spacetime in a way which respects the equivalence principle, but this leaves open a wide range of possible Higgs models. Two types of Higgs inflation take advantage of this ambiguity in the way in which the Higgs couples to gravity.

The oldest type of Higgs inflation includes a coupling between the Higgs field and the Ricci scalar R of the spacetime geometry [20–23]. This interaction is given by a Lagrangian density \mathcal{L}_I with a parameter ξ ,

$$\mathcal{L}_I = \xi R \mathbf{H}^\dagger \mathbf{H}. \quad (15)$$

There are no compelling reasons for omitting a curvature-coupling term like this from the scalar field Lagrangian density (apart from technical difficulty).

The curvature-coupling term couples the metric and the scalar field fluctuations. After these have been separated, the scalar fluctuation amplitude given previously by (11) now becomes [24],

$$A_s = \frac{2}{3\pi^2} \lambda \frac{N_*^3}{(1 + 8\xi N_*)(1 + 6\xi)}. \quad (16)$$

The limits on A_s set by the CMB can now be realised provided that $\xi \sim 10^5$, maybe rather large but not as unreasonable as the previous restrictions on λ .¹

The tensor-scalar ratio r and the spectral index of the scalar fluctuations n_s can be calculated for

¹These calculations usually proceed by using a fictitious metric called the Einstein frame [25]. The results here have been calculated using the physical metric of the Standard Model, or the ‘Jordan frame’.

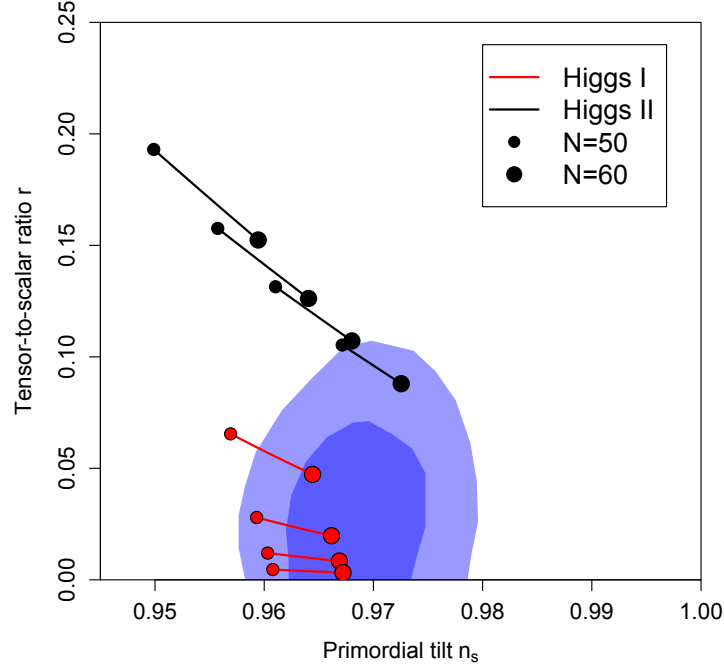


Figure 2. Fluctuations in the CMB can be used to infer the primordial amplitude ratio of tensor to scalar modes r and the spectral index of the scalar fluctuations n_s . The predictions for two types of Higgs inflation are shown, with a selection of ξ and w parameters (see text). The solid areas are the 67% and 95% confidence regions copied from the Planck publication [18], which combines CMB and other large scale structure data to obtain limits on r and n_s .

this model and, if we set aside the question of quantum gravity corrections to the Higgs potential,

$$r_* = \frac{16}{N_*} \frac{1 + 6\xi}{1 + 8\xi N_*} \quad (17)$$

$$n_s = 1 - \frac{3}{N_*} \frac{3 + 16\xi N_*}{3 + 24\xi N_*}. \quad (18)$$

These are plotted in figure 2, which shows how r decreases as ξ ranges from 0.01 to 100. (The plots of $\xi > 100$ are almost identical to the $\xi = 100$ case.) The original Higgs model with $\xi = 0$ gives values for r and n_s which are outside of the range shown by this plot. The agreement with the observational limits for the wide range of values for ξ in this plot is impressive.

This model is not without its problems, however. The large value of ξ increases the effect of quantum gravity corrections to the Higgs potential at values of the Higgs field encountered during inflation [26–30]. It seems, though this is still controversial, that these corrections may affect the values of r and n_s [24, 31]. It also has to be mentioned that there are many other models of inflation which produce agreement with the observed limits on r and n_s [18]. The special feature of Higgs inflation is that it is based on a field which we already know exists.

3.2. Type II Higgs inflation

The interactions between the Higgs field and gravity have one further restriction, and this is a limit on the number of derivative terms appearing in the Lagrangian density. Having too many derivatives runs the risk of producing an unstable theory with no ground state, because some of

the kinetic terms in the energy can become negative. The second type of Higgs inflation is the next simplest which avoids this problem. It includes a coupling between the Higgs field and the Einstein tensor $G^{\mu\nu}$ of the spacetime geometry [32–34]. The interaction is given by a Lagrangian density with a parameter w ,

$$\mathcal{L}_I = -w^2 G^{\mu\nu} (D_\mu \mathbf{H})^\dagger (D_\nu \mathbf{H}), \quad (19)$$

where $D_\mu \mathbf{H}$ is the $SU(2) \times U(1)$ -symmetric derivative of the Higgs field with respect to the spacetime coordinates and μ runs from $0 \dots 4$.

This new type of curvature coupling again changes the amplitude of density fluctuations, replacing the old result (11) by

$$A_s = \frac{1}{24\pi^2} \frac{V_*}{M_p^4} \frac{(1 + Q_*)}{\epsilon_{V_*}}, \quad (20)$$

where $Q_* = w^2 V_*/M_p^2$ reflects the change to the Higgs derivative terms. The relationship between the the slow-roll parameter and the number of e-folds is also modified to $\epsilon_{V_*} \approx (1 + Q_*/3)/N_*$. The tensor-scalar ratio and the spectral index of the scalar perturbations n_s for this model become,

$$r_* = \frac{16}{N_*} \frac{1 + Q_*/3}{1 + Q_*}, \quad (21)$$

$$n_s = 1 - \frac{3}{N_*} \frac{(1 + 5Q_*/3)(1 + Q_*/3)}{(1 + Q_*)^2}, \quad (22)$$

where $Q_* \propto (wN_*)^{2/3}$. These are plotted in figure 2, with values of w ranging from $0.1 M_p^{-1}$ to $100 M_p^{-1}$ (from top to bottom). The agreement with the observational limits is not as good as before, there is still consistency for large w .

4. The electroweak Higgs

The Higgs field plays the central role in the electroweak phase transition at a temperature around 163 GeV and a time around one nanosecond after the big bang. Just prior to this transition, particle interaction timescales are far shorter than the timescale of the evolving universe, and the radiation which fills the universe is, in effect, in perfect thermal equilibrium. What happens next depends on the details of the Higgs potential.

Thermal particle interactions with the Higgs field introduce temperature dependent terms into the Higgs potential. The leading order terms for the Standard Model are (see e.g. [35]),

$$V(\phi, T) \approx V_0(T) + \frac{1}{2}m^2(T)\phi^2 - ET\phi^3 + \frac{1}{4}\lambda(T)\phi^4, \quad (23)$$

where $m(T)$ and $\lambda(T)$ are an effective thermal mass and coupling of the Higgs field, and

$$E = \frac{1}{6\pi v^3} (2m_W^3 + m_Z^3) \approx 0.0064. \quad (24)$$

This potential is shown in figure 3. At a critical temperature T_c the potential has two minima at $\phi = 0$ and $\phi = \phi_c$ with equal potential $V(0) = V(\phi_c)$. Above this critical temperature, the potential has a global minimum at the symmetric point $\phi = 0$. The $SU(2)$ vector bosons, for example, have equal thermal masses. We can think of this as the symmetric phase. At low temperatures, the Higgs

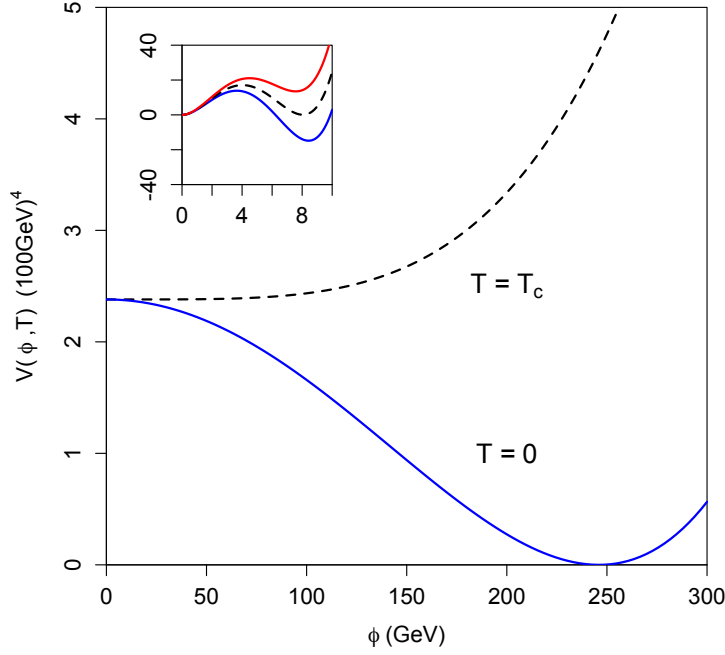


Figure 3. The temperature-corrected Higgs potential implies symmetry restoration at high temperatures. The inset shows the region near the origin, for three temperatures close to $T_c \approx 163$ GeV, and a magnified vertical scale in $(\text{GeV})^4$.

field is close to its vacuum expectation value v , and the particle spectrum is characteristic of the broken symmetry phase.

The potential has a very small barrier near the critical temperature. However, the thermal fluctuations in the Higgs field $\delta\phi \sim T_c$ swamp the barrier, which has width $\phi_c = 2ET_c/\lambda \sim 10^{-2}T_c$ [36]. In this case, as the temperature drops, the thermal average of the Higgs field evolves continuously from the symmetric phase into the broken symmetry phase, maintaining local thermal equilibrium.

The Higgs field also has the three additional components α , β and γ , which eventually become absorbed by the vector potential fields. The initial values of these fields are correlated on small scales but essentially random on large scales and they will evolve to different values in widely separated regions of the universe which are out of causal contact with one another. The electromagnetic field in different parts of the CMB (see Eq. (6)) is initially made up from different combinations of the SU(2) and Higgs fields. The large scale fluctuations in α , β and γ create large scale electric and magnetic field fluctuations, but the field strength is given by a gradient of the Higgs fluctuations and because of the large length scale of the fluctuations the field is too small to be observed.

The continuous electroweak phase transition of the Standard Model seems sadly devoid of any observable consequences. A first order electroweak phase transition, on the other hand, could have significant observational effects such as:

- (1) The generation of baryon asymmetry. Expanding bubbles of broken symmetry phase combine with CP symmetry violation to produce a net excess of baryons over anti-baryons [37–39].
- (2) The generation of gravitational waves. Expanding bubbles of broken symmetry phase act as sources of gravitational waves [40].
- (3) The generation of magnetic fields. Colliding bubbles of broken symmetry phase produce a turbulent dynamo [41].

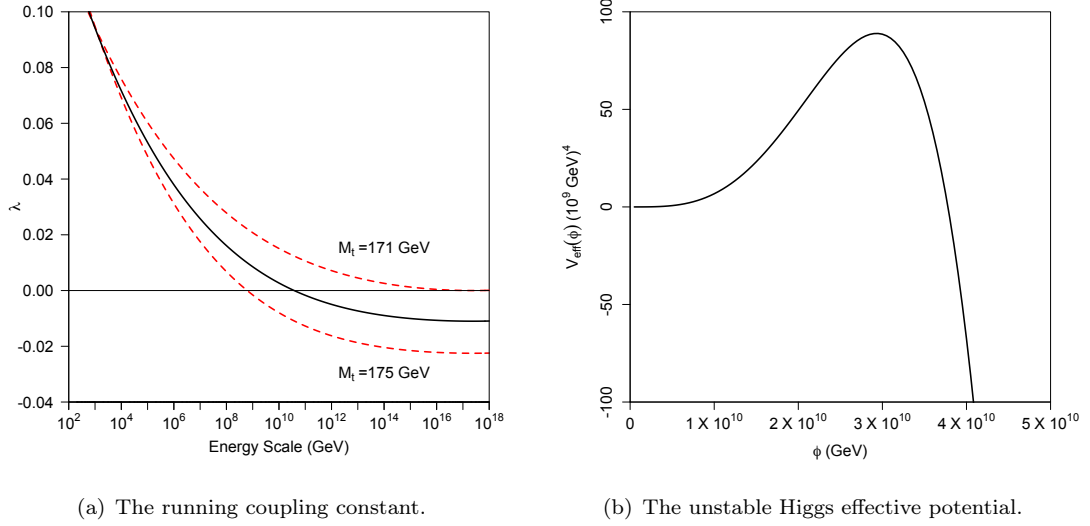


Figure 4. (a) Shows a numerical fit to the running coupling constant for the Higgs field of the standard model with $M_H = 125\text{GeV}$. The running coupling appears to become negative at high energies, resulting in a metastable Higgs potential (b). The coupling is shown for a range of the top quark mass M_t .

Adding extra bosons to the model increases the parameter E in Eq. (24), possibly raising the height of the potential above the magnitude of thermal fluctuations and making the phase transition first order. These extra bosons can come from supersymmetric extensions of the Standard Model [42] and/or additional Higgs fields [43]. Therefore remnants from the electroweak phase transition might possibly provide evidence of new physics at the TeV scale.

5. The unstable Higgs

Our final look at Higgs cosmology considers whether the Higgs field might undergo a phase transition at some time in the future. At the present time the Higgs field would be in a supercooled state, analogous to a bottle of pure water cooled a few degrees below zero Celsius in the freezer. If the water is gently removed from the freezer it sometimes continues in a supercooled liquid state. Any violent disturbance to the bottle causes a rapid transition from water to ice.

The question of Higgs metastability arises even in the Standard Model [8]. Quantum effects modify the Higgs potential, producing an effective potential V_{eff} . At large field values, the potential can be expressed in terms of an effective quartic coupling $\lambda_{\text{eff}}(\phi)$,

$$V_{\text{eff}}(\phi) \approx \frac{1}{4} \lambda_{\text{eff}}(\phi) \phi^4. \quad (25)$$

If λ_{eff} becomes negative, then the state with $\langle \phi \rangle = v$ is no longer the lowest energy state. This state is a ‘false vacuum’ state, which can be at best metastable.

The stability of the Higgs potential is sensitive to the Standard Model parameters, especially the Higgs mass and the top quark mass (which being the most massive fermion is the one with the largest coupling to the Higgs field). In first-order perturbation theory, each bosonic mode with frequency ω contributes a vacuum energy $\hbar\omega/2$ to the potential, and each fermionic mode contributes $-\hbar\omega/2$, due to the fermionic particle statistics. This effect of this sign difference persists to higher orders in perturbation theory, so that a large top quark mass has a destabilising effect on the Higgs potential.

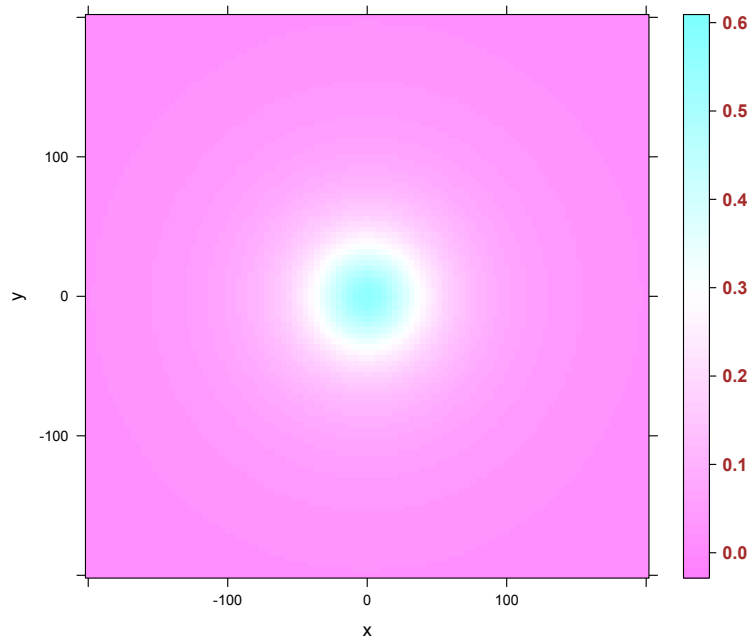


Figure 5. The spatial distribution of the Higgs field in a vacuum decay bubble. Two dimensions are shown, the bounce solution has the same profile in all four dimensions (three space and one imaginary time). The central region of the bubble has large values of ϕ stretching well beyond the potential barrier from the false vacuum (pink) into a new Higgs phase (blue). All measurements are in reduced Planck units. The effective coupling here is modelled by $\lambda_{\text{eff}} = -0.02 + 10^{-4}(\ln \phi/M_p)^2$ and the instanton action is calculated numerally to be $B \approx 1360$.

Figure 4 shows the running coupling $\lambda(\mu)$ in second-order perturbation theory [44]. The running coupling $\lambda(\mu)$ is closely related to the effective coupling, very roughly $\lambda_{\text{eff}}(\phi) \sim \lambda(\phi)$ with a quantifiable correction. The running coupling becomes small at large values of the Higgs field, and runs negative for a Higgs mass $m_H = 125 \text{ GeV}$ and top quark masses above 171 GeV . Measurements of the top quark mass, $173.34 \pm 0.76 \text{ GeV}$ [45], place it in firmly the unstable regime. The effective potential (25) for this range of parameters is also shown in Figure 4.

The unstable false vacuum can decay by quantum tunnelling through the potential barrier, producing bubbles filled with Higgs field at a potential lower than the energy of the false vacuum. The rate of bubble nucleation is determined by a procedure adapted from the theory of supercooled first order phase transitions. Recall that, for phase transitions, the probability of bubble nucleation is proportional to $e^{-\Delta S}$, where ΔS is the change in entropy due to the bubble. For vacuum decay, one first solves the field equations with an imaginary time coordinate $\tau = it$, to obtain a solution which Coleman called the ‘bounce’ [46–48]. The bubble nucleation rate is given by

$$\Gamma_D = Ae^{-B}, \quad A \approx \frac{B^2}{4\pi^2 r_b^4}, \quad (26)$$

where B , the action of the bounce solution, plays the same role as the entropy ΔS in a phase transition. The pre-factor A is related to the bubble radius r_b . The Higgs field profile of a bubble after it nucleates matches the $\tau = 0$ slice through the bounce solution. After nucleation, these bubbles are unstable and they grow, eventually expanding at the speed of light. The interior of the bubble sinks towards the true Higgs vacuum, and the spacetime geometry inside becomes wildly

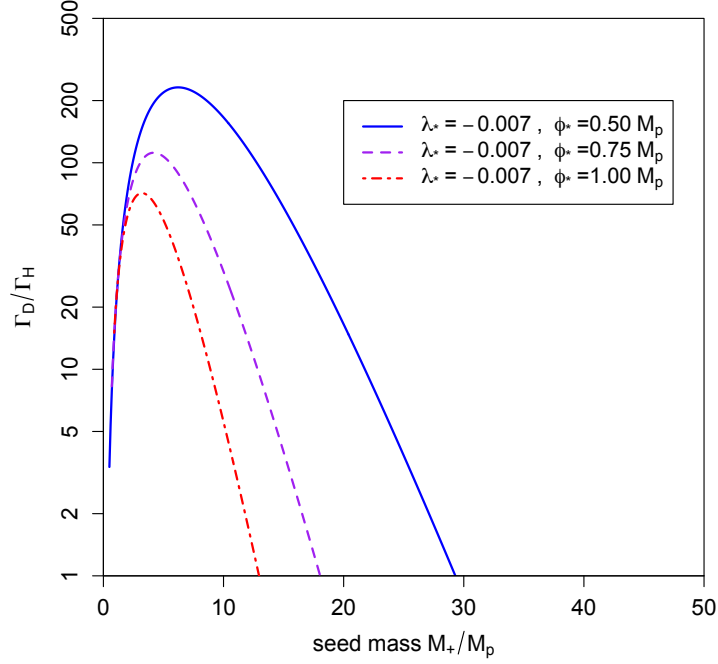


Figure 6. The ratio of the rate of vacuum decay rate caused by a black hole Γ_D to the Hawking evaporation rate of the black hole Γ_H for different black hole masses M_+ . The effective coupling has been modeled by $\lambda_{\text{eff}} = \lambda_* + 10^{-4}(\ln \phi/\phi_*)^2$, and the Hawking evaporation rate here is $\Gamma_H \approx 3.6 \times 10^{-4} M_P^4/M_+^3$. For these model parameters, a primordial black hole seeds vacuum decay after it reaches a mass $10 - 30 M_P$.

distorted, whilst outside the expanding bubble eats into the false vacuum state.

An example of a bubble solution to the coupled Higgs and gravitational field equations is shown in Figure 5. A single bubble of this type would eventually switch the vacuum state of the observable universe. Luckily, the nucleation rate for this type bubble is entirely negligible. Using formula (26), the pre-factor for the bounce solution shown in Figure 5 is $A \sim 10^{69} \text{ GeV}^4$, but the action $B \approx 1320$ and the exponential term in the nucleation rate dominates.

On the whole, vacuum decay rates tend to be very strongly exponentially suppressed. However, in nature most phase transitions take place due to some type of nucleation seed, an impurity or an imperfection in a containment vessel. There has been some discussion recently as to whether a tiny black hole could act as a seed and trigger false vacuum decay [49]. Small black holes may be produced in the early universe [50], or in a high energy accelerator [51–53].

One obstacle to black holes as nucleation seeds is that black holes evaporate by the Hawking process, and small black holes evaporate very rapidly. Therefore it is important to compare the seeded nucleation rate to the rate of Hawking evaporation. This has been done in Figure 6, where we see that the vacuum decay rate beats the Hawking evaporation rate for small holes with masses 10–100 times the reduced Planck mass.

If we start with a primordial black hole of mass around 10^{12} kg produced in the early universe, it takes a few Gyr before it decays to the relevant mass range. At this point, it can nucleate the vacuum decay bubble and initiate the phase transition. Clearly, no such phase transition has taken place, therefore the metastable Higgs vacuum is inconsistent with the existence of even a single primordial black in our observable universe. This is puzzling, because the standard model Higgs potential produces an unstable Higgs potential with the presently measured values of the top quark and Higgs boson mass. The resolution may be that there are stabilising contributions to the Higgs

potential from physics beyond the standard model.

There is another way in that tiny black holes can occur, which requires the existence of extra dimensions. In the relevant theoretical framework, the Standard Model particles lie on a surface, or ‘brane’, in higher dimensions. The higher-dimensional Planck mass has a similar scale to the Standard Model, but becomes exponentially enlarged on our brane. Black holes can then be produced in particle collisions with energy far below the three-dimensional Planck mass, possibly even at the scale of the Large Hadron Collider. This raises an alarming prospect of triggering vacuum decay at the LHC, but fortunately, we have some re-assurance from the fact that cosmic ray collisions have occurred in nature at energies higher than those that can be reached at the LHC [54]. Although ‘head on’ cosmic ray proton collisions with energy in the TeV range or larger are incredibly rare by particle collider standards, they are quite common over cosmological length and time scales relevant for cosmological vacuum decay. Cosmic ray collisions have not triggered vacuum decay, and neither should the LHC. Once again, microscopic black holes are incompatible with unstable Higgs potentials.

6. Outlook

Our short tour of Higgs cosmology has been largely confined to the the particle content of the Standard Model plus gravity. We have not said anything about the possibilities of new physics in the desert between 100 GeV and the scale of Grand Unification, or indeed about Higgs fields associated with Grand Unification. Some of the ideas mentioned here can be carried across directly to the GUT Higgs field. Models of Higgs inflation work equally well with GUT Higgs fields, and the flexibility in the Higgs potential allows a wider range of inflationary scenarios.

We can be confident, at the very least, that quantum gravity corrections will have an effect both on the Higgs inflationary scenarios and the Higgs stability question. Since we are working up from low energies towards the Planck scale, it seem likely that perturbative quantum gravity is a good place to begin [55]. This speculative end of Higgs physics is therefore providing an impetus for new developments in quantum gravity. Finally, Higgs cosmology offers the possibility that we shall see observational clues, gravity waves from the electroweak phase transition or the detection of primordial black holes, which point the way to physics beyond the standard model.

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